INTRODUCTION

Power system design always requires information of line constants and parameters such as impedance, inductance and capacitance. Those are particularly essential for relay engineers or power protection designers when they set a relay or design a protection scheme. However, incorrect information of line constants can sometimes wrongly affect the whole protection system or cause protection fails in power systems. In order to avoid any errors caused by missing line parameters or incorrect information, designers or engineers frequently need to calculate them. This calculation can offer sequence impedance value as well as phase impedance. The effectiveness of the calculation will be introduced in this report with an example of unbundled 3 phase and ground cable structure. The list of explanations for the line constants calculation is shown as below:

- Information of Conductors for calculation of Line Parameter
- Dimension at a Line Pole
- Calculation of Line Constants (Assumed No Ground Wire)
- Calculation of Line Constants (With Ground Wire)
- Line Constant Program Comparison

INFORMATION OF CONDUCTORS FOR CALCULATION OF LINE PARAMETER

Information of conductors should be ready first before line constants calculation is performed. The elements which are required for calculation are as follows:

- Diameter of phase conductor
- Diameter of ground wire
- Conductor’s operating temperature
- Resistances of conductor by temperature (Ω/mile)
- Frequency of power system
- Distance between phase
If the conductor’s type and producer are known, the information above could be easily found in producers’ website. Most cable production companies offer their product’s information similar to the table below. The distance between phases will be explained more specifically in the next paragraph.

<table>
<thead>
<tr>
<th>Code Word</th>
<th>Cross Sectional Area (in²)</th>
<th>Diameter (in)</th>
<th>Weight (pounds/mile)</th>
<th>Tension (kip)</th>
<th>Break Strength (kip)</th>
<th>Ambiguity (X)</th>
</tr>
</thead>
<tbody>
<tr>
<td>15/1</td>
<td>0.0086</td>
<td>0.205</td>
<td>0.029</td>
<td>0.005</td>
<td>0.005</td>
<td>0.001</td>
</tr>
<tr>
<td>15/2</td>
<td>0.0156</td>
<td>0.217</td>
<td>0.031</td>
<td>0.007</td>
<td>0.007</td>
<td>0.001</td>
</tr>
<tr>
<td>15/3</td>
<td>0.0226</td>
<td>0.229</td>
<td>0.033</td>
<td>0.009</td>
<td>0.009</td>
<td>0.001</td>
</tr>
<tr>
<td>15/4</td>
<td>0.0296</td>
<td>0.241</td>
<td>0.035</td>
<td>0.011</td>
<td>0.011</td>
<td>0.001</td>
</tr>
<tr>
<td>15/5</td>
<td>0.0366</td>
<td>0.253</td>
<td>0.037</td>
<td>0.013</td>
<td>0.013</td>
<td>0.001</td>
</tr>
</tbody>
</table>

**Figure 1. Conductor information (ACSS/TW) produced by Southwire LLC.**

**DIMENSION AT A LINE POLE**

The distances between conductors will be absolutely necessary for calculation because an electromagnetic field is one of the elements that can affect impedance and it is changed by distance among conductors.
The distances between conductors can be easily found through calculating the hypotenuse of a right-angled triangle similar to the example above which came from pole structure drawing.

**GMR**

GMR stands for ‘Geometric Mean Radius’. The radius of Phase conductor is 1.108/2=0.554 as shown in Figure.1 table. Since each phase is not bundled and has the same kind of conductor, GMR is calculated as below:

\[
GMR_a = GMR_b = GMR_c = \text{Radius of Phasor Conductor} \times e^{-\frac{1}{4}} \text{ in} \\
= 0.554 \times e^{-1/4} \text{ in} = 0.4315 \text{ in} = 0.036 \text{ ft}
\]

Ground wire’s radius is 0.2955 and GMR of ground wire is as follows:

\[
GMR_g = 0.2955 \times e^{-1/4} \text{ in} = 0.2301 \text{ in} = 0.0192 \text{ ft}
\]

**GMD**

GMD stands for ‘Geometric Mean Distance’ and the method to get it is explained as below:

\[
D_{ab} = 15.977 \text{ ft} \\
D_{bc} = 16.919 \text{ ft} \\
D_{ca} = 11.045 \text{ ft}
\]

\[
GMD = (D_{ab} \times D_{bc} \times D_{ca})^{1/3} = (15.977 \times 16.919 \times 11.045)^{1/3} = 14.339 \text{ ft}
\]
CALCULATION OF LINE CONSTANTS (ASSUMED NO GROUND WIRE)

More commonly, real power systems include ground wire but this should be pre-comprehended as a basic concept of calculation self-impedance, and mutual impedance.

**SELF-IMPEDANCE**

\[
Z_{aa} = Z_{bb} = Z_{cc} = (r_a + r_d) + j\omega k ln \frac{D_e}{GMR_a} = 0.1127 + 0.09528 + j0.12134 \times ln \frac{2790}{0.036} = 0.20798 + j1.366 \ \Omega/\text{mi}
\]

\[
(D_e = 2160 \sqrt{\frac{\rho}{f}} = 2160 \sqrt{\frac{100}{60}} = 2790 \text{ ft})
\]

**MUTUAL-IMPEDANCE**

\[
Z_{ab} = r_d + j\omega k ln \frac{D_e}{D_{ab}} = 0.09528 + j0.12134 \times ln \frac{2790}{15.977} = 0.09528 + j0.6264 \ \Omega/\text{mi}
\]

\[
Z_{bc} = r_d + j\omega k ln \frac{D_e}{D_{bc}} = 0.09528 + j0.12134 \times ln \frac{2790}{16.919} = 0.09528 + j0.6195 \ \Omega/\text{mi}
\]

\[
Z_{ac} = r_d + j\omega k ln \frac{D_e}{D_{ac}} = 0.09528 + j0.12134 \times ln \frac{2790}{11.045} = 0.09528 + j0.6712 \ \Omega/\text{mi}
\]

**INDUCTANCE**

\[
\text{Inductance} = 1.366 \ \Omega/\text{mi} = 2.252 \times 10^{-6} \text{ H/m}
\]

**CAPACITANCE**

\[
\bar{C}_a = \bar{C}_b = \bar{C}_c = \bar{C} = \frac{2\pi \varepsilon}{\ln(D_m/R_b)} = \frac{2\pi \times 8.854 \times 10^{-12}}{\ln(14.399/0.036)} = 9.2852 \times 10^{-12} \text{ F/m}
\]

**SUSCEPTANCE & SHUNT CAPACITIVE REACTANCE**

\[
B = w\bar{C} = 2\pi \times 60 \times 9.2852 \times 10^{-12} = 3.5 \times 10^{-9} \text{ mho/m} = 5.6322 \times 10^{-6} \text{ mho/\text{mi}}
\]

**SEQUENCE IMPEDANCE**

\[
Z_{abc} = \begin{bmatrix}
0.20798 + j1.366 & 0.09528 + j0.6264 & 0.09528 + j0.6712 \\
0.09528 + j0.6264 & 0.20798 + j1.3669 & 0.09528 + j0.6195 \\
0.09528 + j0.6712 & 0.09528 + j0.6195 & 0.20798 + j1.366
\end{bmatrix} \ \Omega/\text{mi}
\]

\[
Z_{012} = \frac{1}{3} \begin{bmatrix}
1 & 1 & 1 \\
1 & \alpha & \alpha^2 \\
1 & \alpha^2 & \alpha
\end{bmatrix} \begin{bmatrix}
Z_{aa} & Z_{ab} & Z_{ac} \\
Z_{ab} & Z_{bb} & Z_{bc} \\
Z_{ac} & Z_{bc} & Z_{cc}
\end{bmatrix} \begin{bmatrix}
1 & 1 & 1 \\
1 & \alpha & \alpha^2 \\
1 & \alpha^2 & \alpha
\end{bmatrix}
\]
CALCULATION OF LINE CONSTANTS (WITH GROUND WIRE)

If lines have ground wires, the total impedances of conductors would be changed as below because mutual impedances between phase conductors and ground wires should be considered.

\[
Z_{ww} = (r_w + r_d) + j \omega k \ln \frac{D_e}{GMR_g} = (0.7049 + 0.09528) + j0.12134 \times \ln \frac{2790}{0.0192} = 0.8002 + j1.14423 \ \Omega/\text{mi}
\]

\[
Z_{aw} = r_d + j\omega kln \left( \frac{D_e}{D_{aw}} \right) = 0.09528 + j0.12134 \times \ln \frac{2790}{14.185} = 0.09528 + j0.6409 \ \Omega/\text{mi}
\]

\[
Z_{bw} = r_d + j\omega kln \left( \frac{D_e}{D_{bw}} \right) = 0.09528 + j0.12134 \times \ln \frac{2790}{19.076} = 0.09528 + j0.6049 \ \Omega/\text{mi}
\]

\[
Z_{cw} = r_d + j\omega kln \left( \frac{D_e}{D_{cw}} \right) = 0.09528 + j0.12134 \times \ln \frac{2790}{24.558} = 0.09528 + j0.5743 \ \Omega/\text{mi}
\]

\[
Z_{abc} = \begin{bmatrix}
Z_{aa} - \frac{Z_{aw}Z_{aw}}{Z_{ww}} & Z_{ab} - \frac{Z_{aw}Z_{bw}}{Z_{ww}} & Z_{ac} - \frac{Z_{aw}Z_{cw}}{Z_{ww}} \\
Z_{ba} - \frac{Z_{aw}Z_{bw}}{Z_{ww}} & Z_{bb} - \frac{Z_{bw}Z_{bw}}{Z_{ww}} & Z_{bc} - \frac{Z_{bw}Z_{cw}}{Z_{ww}} \\
Z_{ca} - \frac{Z_{aw}Z_{cw}}{Z_{ww}} & Z_{cb} - \frac{Z_{bw}Z_{cw}}{Z_{ww}} & Z_{cc} - \frac{Z_{cw}Z_{cw}}{Z_{ww}}
\end{bmatrix} \Omega/\text{mi}
\]

\[
Z_{abc} = \begin{bmatrix}
0.2614 + j1.1171 & 0.1437 + j0.3908 & 0.1395 + j0.4468 \\
0.1437 + j0.3908 & 0.2518 + j1.1429 & 0.1352 + j0.4071 \\
0.1395 + j0.4468 & 0.1352 + j0.4071 & 0.2443 + j1.1638
\end{bmatrix} \Omega/\text{mi}
\]

LINE CONSTANT PROGRAM COMPARISON

The result of the example calculation above can be checked by comparing it to the result from the line constant program. In order to make this comparison, CAPE program was used. In order to run this program, the actual line length should be inserted and this amount is 3.64 miles. The results between the hand calculation and the CAPE line constant program are almost identical.

ACTUAL LINE IMPEDANCE BY HAND CALCULATION

\[
Z_{abc} = \begin{bmatrix}
0.2614 + j1.1171 & 0.1437 + j0.3908 & 0.1395 + j0.4468 \\
0.1437 + j0.3908 & 0.2518 + j1.1429 & 0.1352 + j0.4071 \\
0.1395 + j0.4468 & 0.1352 + j0.4071 & 0.2443 + j1.1638
\end{bmatrix} \Omega \times \frac{1}{\text{mi}} \times 3.64 \text{ mi}
\]
THE RESULT FROM CAPE PROGRAM

\[
Z_{abc} = \begin{bmatrix}
0.9515 + j4.0662 & 0.5231 + j1.4225 & 0.5078 + j1.6264 \\
0.5231 + j1.4225 & 0.9166 + j4.1602 & 0.4921 + j1.4818 \\
0.5078 + j1.6264 & 0.4921 + j1.4818 & 0.8893 + j4.2362
\end{bmatrix} \Omega
\]

About Authors
Seunghwa Lee holds a Bachelor’s of Science degree in Electrical Engineering from Seoul National University, in South Korea. He also received a Masters of Engineering degree in Electrical Engineering from Texas A&M University with concentration in Power Systems. In 2014, Seunghwa became a registered professional engineer in the State of Texas (License No.118682). He is currently working on all projects related to power protection such as: protection design, relay setting, and forensic analysis of fault event records with proficiency in ASPEN and CAPE simulator.

Joe Perez holds a Bachelor’s of Science degree in Electrical Engineering from Texas A&M University and is a registered professional electrical engineer in the state of Texas. He established SynchroGrid to provide electric utilities with power system protection applications, analysis, and training. Joe has provided services to relay manufacturers, utilities, and oil and gas companies. He has in-depth experience in power system protection settings and design from the manufacturer and utility perspectives. He has written papers for major relay conferences in the U.S. and many application notes and testing procedures for relay manufacturers and utilities. He is also the co-creator of a transformer rating software.

About SynchroGrid
SynchroGrid is a Texas-based company that offers electric utilities with professional consulting services, including transmission line, substation, and system protection design, relay settings development, relay coordination studies, and relay mis-operation analyses.

For sales and general questions, please contact us:

SynchroGrid, LLC
Attn: Seunghwa Lee
4030 Highway 6 South, STE 325
College Station, Texas 77845
Phone: (952) 446-5443
Email: shlee1564@synchrogrid.com

SynchroGrid, LLC
Attn: Joe Perez
4030 Highway 6 South, STE 325
College Station, Texas, 77845
Phone: (713) 471-3429
Email: jperez@synchrogrid.com